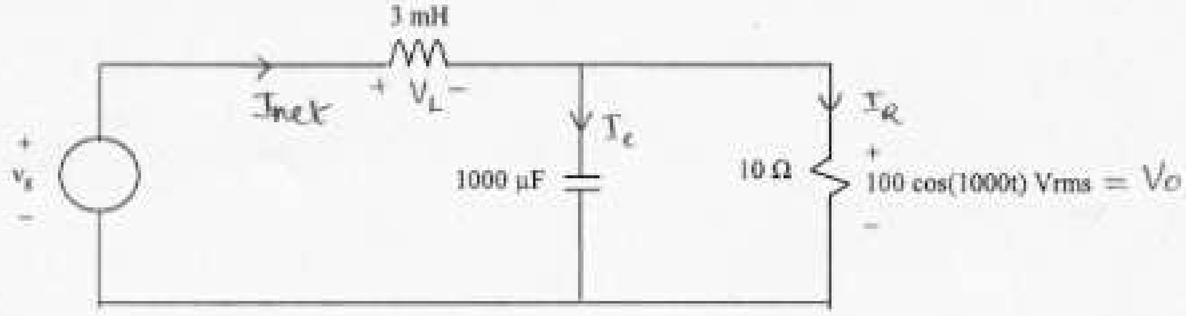
Q #1 (25 pts) **Phasors**

Use phasor techniques to find $v_g(t)$.



$\omega = 1000$ rps

$Z_L = j\omega L = j3\ \Omega$

$V_g = V_L + V_o$

$Z_C = \dfrac{-j}{\omega C} = \dfrac{-j}{1000\times 10^{-6}\times 1000} = -j\ \Omega$

$I_R = \dfrac{V_o}{R} = \dfrac{100}{10} = 10\ A$

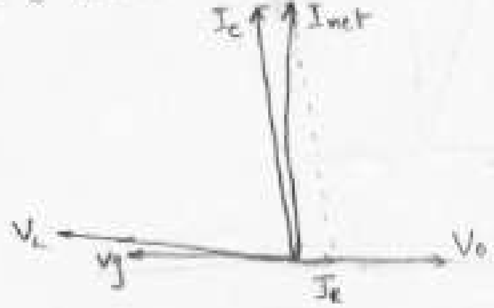
$I_c = \dfrac{V_o}{Z_c} = \dfrac{100}{-j} = j100\ A = 100\angle 90^\circ\ A$

$I_{net} = I_R + I_c = 10 + j100 = 100.5\angle 84.28^\circ\ A$

$V_L = (I_{net})\ j3\ \Omega = (10 + j100)\ j3 = j30 - 300 = 301.5\angle 174.28^\circ$

$V_g = V_L + V_o = 100 + j30 - 300 = -200 + j30 = 202.23\angle 171.46^\circ$

$\therefore v_g = 202.23\cos(1000t + 171.46^\circ)$



Q #2 (25 pts) **Phasors**

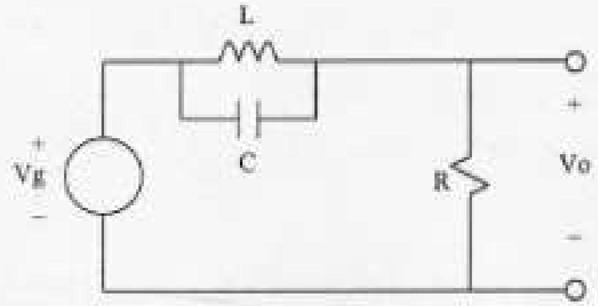
Vg is sinusoidal. The following questions refer to sinusoidal steady-state operation.

a) At what frequency (ω) is Vo zero?

As ω approaches zero or infinity, the ratio Vo/Vg approaches specific values.

b) What value does Vo/Vg approach as ω approaches zero?
c) What value does Vo/Vg approach as ω approaches infinity?

Express your answers in terms of R, L, and C.



$$Z_{net} = R + \frac{j\omega L \cdot \frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}} = R + \frac{j\omega L}{1-\omega^2 LC} = \frac{R(1-\omega^2 LC) + j\omega L}{1-\omega^2 LC}$$

a) $\frac{\bar{V}_o}{\bar{V}_g} = \frac{R}{R + Z_L \| Z_C}$; $Z_L \| Z_C = -\frac{X_L X_C}{j(X_L + X_C)}$

$\frac{\bar{V}_o}{\bar{V}_g} = \frac{R}{R + \frac{-X_L X_C}{j(X_L+X_C)}} = j\frac{R(X_L + X_C)}{-X_L X_C + jR(X_L + X_C)}$; $\bar{V}_o = 0$ when $X_L + X_C = 0$ ie $\omega L = \frac{1}{\omega C}$, $\therefore \omega = \frac{1}{\sqrt{LC}}$

b) $\frac{V_o}{V_g} = \frac{R}{R + (Z_L \| Z_C)} = \frac{R}{R + \frac{j\omega L \frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}}} = \frac{R}{R + \frac{j\omega L}{-\omega^2 LC + 1}} = \frac{R(1-\omega^2 LC)}{j\omega L + R(1-\omega^2 LC)}$

When $\omega \to 0$, $\frac{V_o}{V_g} = 1$

c) When $\omega \to \infty$, by using L'Hopital identity and taking $\lim_{\omega\to\infty}$

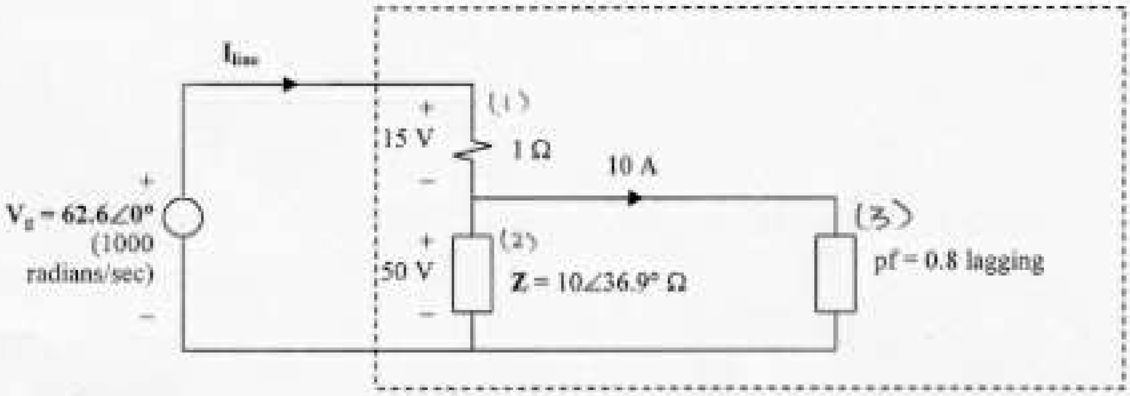
$$\lim_{\omega\to\infty}\left\{\frac{\cancel{\omega^2}(R)(\underbrace{\tfrac{1}{\omega^2}}_{=0} - LC)}{\cancel{\omega^2}\left(\underbrace{\tfrac{jL}{\omega}}_{=0} + R\left(\underbrace{\tfrac{1}{\omega^2}}_{=0} - LC\right)\right)}\right\} = \frac{R(-LC)}{R(-LC)} = 1$$

$\therefore \frac{V_o}{V_g} = 1$

Q #3 (25 pts) Power

a) **Find the power factor of the total load.**
b) **Find the phasor value I_{line} (magnitude and angle).**
c) **Find the equivalent impedance of the total load.**

[All voltages and currents are rms values]
[Unless specified, the phasor angles of the given voltages and currents are not known and should not be assumed to be zero]



TOTAL LOAD

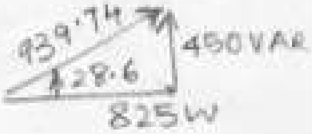
$V_1 = 15V$
$Z_1 = 1\Omega$
$\therefore P_1 = \frac{V_1^2}{Z_1} = 225W$

$V_2 = 50V$
$Z_2 = 10\angle 36.9^\circ$
$= 7.996 + j6.0$
$S_2 = \frac{V_2^2}{Z_2^*} = \frac{50^2}{7.996 - j6}$
$= 200 + j150$
$= 250 \angle +36.88^\circ$
$\therefore P_2 = 200\ W$
$Q_2 = +150 VAR$

$V_3 = 50V$
$pf = 0.8$ lagging
$I_3 = 10A$
$S_3 = V_3 \cdot I_3 = 500VA$

$P_3 = 400W$
$Q_3 = 300 VAR$

$\therefore P_{net} = P_1 + P_2 + P_3 = 225 + 200 + 400 = 825W$
$Q_{net} = Q_2 + Q_3 = 300 + 150 = 450 VAR$



(a) $pf = \frac{825}{939.74} = 0.8778$ lagging

(b) $I_{line} = \frac{939.74}{62.6} = 15.01 \angle -28.6$

Calculation page

(c) Equivalent impedance of the load:

$S_{net} = P_{net} + j\,Q_{net} = 825 + j450 = 939.74 \angle 28.6^\circ$ VA

$Z_{net} = \dfrac{S_{net}}{I_{line}^2} = \dfrac{939.74 \angle 28.6}{15.01^2}$

$= 4.171 \angle 28.6^\circ$

Q #4 (25 pts) Power

Two loads are attached to a 60 Hz (377 radians/sec), 480 Vrms source.

Load 1:	5 kW,	pf: 0.6 lagging
Load 2:	3 kVA,	pf: 0.9 leading

Your job is to boost the power factor of the combined load to 0.9.

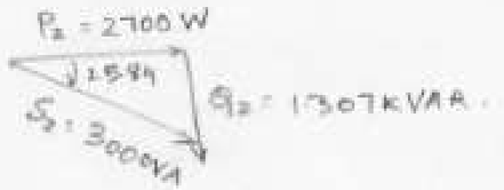
a) **What is the original power factor of the combined load (load 1 and load 2 together)?**

b) **How would you increase the combined power factor to 0.9?**
Be specific - What would you use? Where would you put it? Values?

Load 1 :

$\theta = \cos^{-1} 0.6$
$= 53.13°$

Load 2 :



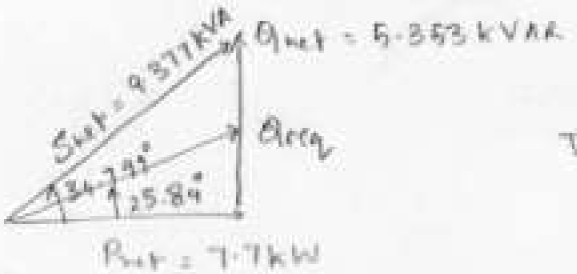
$\theta = \cos^{-1} 0.9$
$= 25.84$

a) pf of combined load :

$$pf = \frac{7.7 \times 10^3}{9.377 \times 10^3} = 0.821$$

$\theta = 34.799°$

(b)



To increase pf $= 0.9$,
$\theta_{new} = 25.84°$

$$\Delta Q = 5353 - 7700 \tan 25.84° = 1623.7 \text{ VAR}$$

$$1623.7 = \frac{V^2}{X_c} = \omega C V^2$$

$\therefore C = 18.69$ µF.