Q #1 (25 pts) **Laplace**

Given . . .

f (t) = 5 e^{-3t} cos(4t) u(t), and

g (t) = $\int_0$ f(τ) dτ + f (t)

Derive g(t) using Laplace techniques.

$$F(s) = \frac{5(s+3)}{(s+3)^2+16} = \frac{5s+15}{s^2+6s+25}$$

$$G(s) = \frac{F(s)}{s} + F(s) = \frac{5s+15}{s(s^2+6s+25)} + \frac{5s+15}{s^2+6s+25}$$

$$G(s) = \frac{5s+15+5s^2+15s}{s(s^2+6s+25)} = \frac{5s^2+20s+15}{s(s^2+6s+25)}$$

$$G(s) = \frac{A}{s} + \frac{Bs+C}{s^2+6s+25}$$

$$A = s\,G(s=0) = \frac{15}{25} = 0.6$$

To Find B + C, clear fractions + equate powers of s.

$$5s^2+20s+15 = 0.6s^2+3.2s+15+Bs^2+Cs$$

s^2: $5 = 0.6 + B \rightarrow B = 4.4$

s^1: $20 = 3.2 + C \rightarrow C = 16.4$

s^0: $15 = 15$ ✓

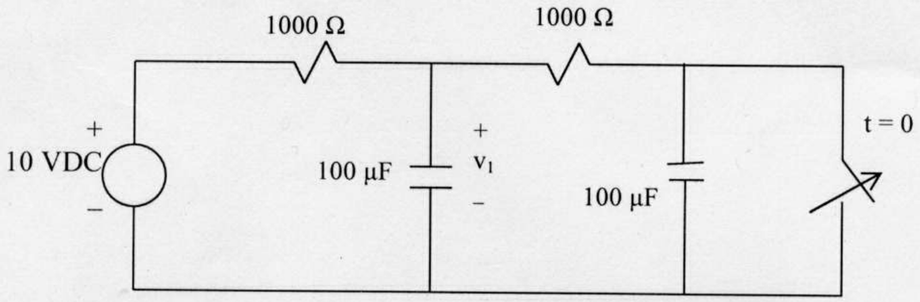
$$G(s) = \frac{0.6}{s} + \frac{4.4s+16.4}{s^2+6s+25} = \frac{0.6}{s} + \frac{4.4(s+3)}{(s+3)^2+4^2} + \frac{0.8(4)}{(s+3)^2+4^2}$$

$$g(t) = (0.6 + 4.4e^{-3t}\cos 4t + 0.8e^{-3t}\sin 4t)\,u(t)$$

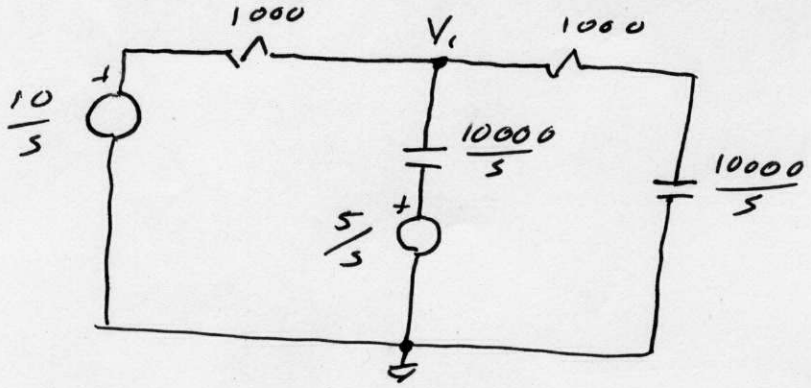
Q #2 (25 pts) Laplace Circuit Analysis

The circuit shown has been in steady state for a long time before the switch opens at t=0.

Find $v_1(t)$ using Laplace methods.



$v_1(0^-) = 5\,V$



KCL @ V1

$$\frac{V_1 - 10/s}{1000} + \frac{V_1 - 5/s}{10000/s} + \frac{V_1}{1000 + 10000/s} = 0$$

$$V_1\left(\frac{1}{1000} + \frac{s}{10000} + \frac{1}{1000 + 10000/s}\right) = \frac{1}{100s} + \frac{1}{2000}$$

$$V_1\left(1 + \frac{10}{s} + \frac{s}{10} + 1 + 1\right) = \frac{10}{s} + \frac{100}{s^2} + \frac{1}{2} + \frac{5}{s}$$

$$V_1 = \frac{0.5 + 15/s + 100/s^2}{s/10 + 3 + 10/s} = \frac{0.5s^2 + 15s + 100}{s(0.1s^2 + 3s + 10)}$$

Calculation page

$V_1 = \dfrac{\cancel{0.5 s^2 + 15 s + 100}}{\cancel{s(0.1 s^2}}$

$$V_1 = \frac{5s^2 + 150s + 1000}{s(s^2 + 30s + 100)} = \frac{5s^2 + 150s + 1000}{s(s+3.82)(s+26.2)}$$

$$V_1 = \frac{A}{s} + \frac{B}{s+3.82} + \frac{C}{s+26.2}$$

$$A = sV_1(0) = \frac{1000}{(3.82)(26.2)} = \cancel{9.99}\ 10$$

$$B = (s+3.82)V_1(-3.82) = \frac{72.96\cancel{4} - 573 + 1000}{(-3.82)(22.38)}$$

$$B = -5.85$$

$$C = (s+26.2)V_1(-26.2) = \frac{3432 - 3930 + 1000}{(-26.2)(-22.38)}$$

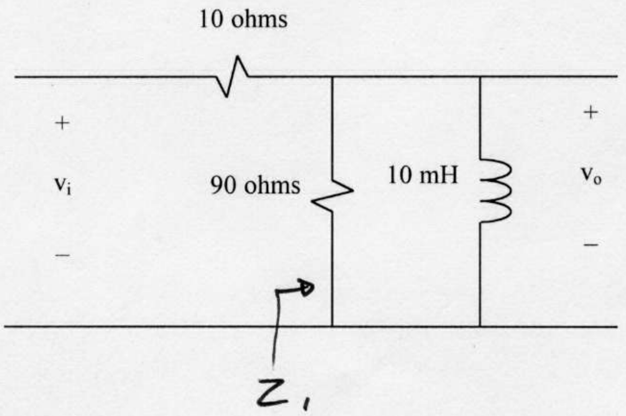
$$C = 0.856$$

$$V_1 = \frac{\cancel{9.99}\ 10}{s} + \frac{-5.85}{s+3.82} + \frac{0.856}{s+26.2}$$

$$v_1(t) = \left(\cancel{9.99}\ 10 - 5.85e^{-3.82t} + 0.856e^{-26.2t}\right)u(t)$$

Q #3 (25 pts) Filters

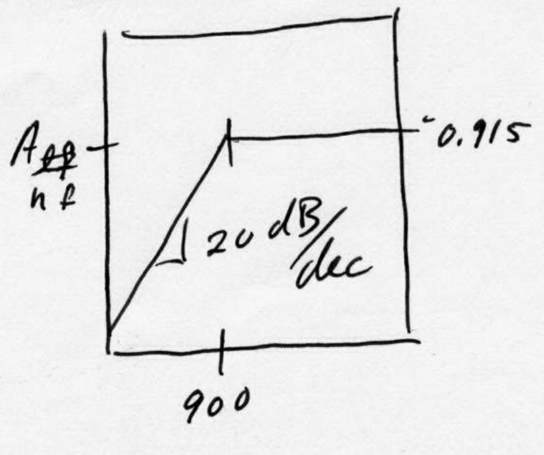
a. Derive the transfer function of this circuit..
b. Plot the Bode diagram of this transfer function (straight line approximation, magnitude only).



$$Z_1 = 90 \| 0.01s = \frac{0.9s}{0.01s+90} = \frac{90s}{s+9000}$$

$$H = \frac{Z_1}{Z_1+10} = \frac{90s}{(s+9000)\left(\frac{90s}{s+9000}+10\right)} = \frac{90s}{90s+10s+90000}$$

$$H = \frac{90s}{100s+90000} = \frac{0.9s}{s+900}$$



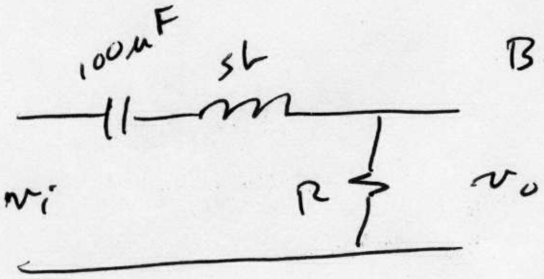
$A_{hf} = 0.9 = -0.915\,dB$

Q #4 (25 pts) Filters

Using 100 uF capacitors, design a band-pass filter and a band-reject filter, both with the following specifications.

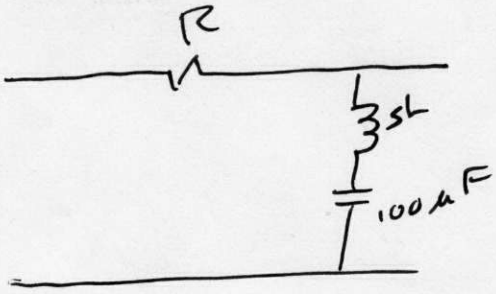
Center frequency: 1000 radians/second
Bandwidth: 100 radians/second
Unity gain at the center frequency

Draw the circuits and specify the component values.



Band pass

R = 1Ω
L = 10mH



Band reject