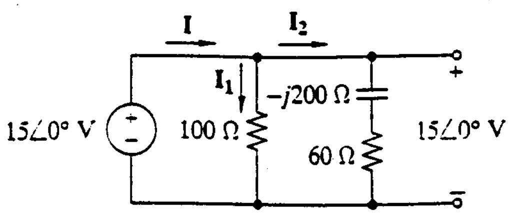
Problem 1:

For the circuit below:

A) Calculate the complex power absorbed by each of the two parallel branches.

B) Calculate the complex power developed by the source and the power factor of the load seen by the source.



A] $\underline{I}_1 = \dfrac{\underline{V}}{100\Omega} = \dfrac{15\angle 0°}{100} = (0.15\angle 0°)$ A

$S_1 = \frac{1}{2}\underline{V}\cdot\underline{I}_1^* = (1.125\angle 0°)$ VA

$\underline{I}_2 = \dfrac{\underline{V}}{(60-j200)\Omega} = \dfrac{15\angle 0°}{208.8\angle -73.3°} = (71.84\angle 73.3°)$ mA

$S_2 = \frac{1}{2}\underline{V}\cdot\underline{I}_2^* = (0.539\angle -73.3°)$ VA

B] $S_{source} = -\frac{1}{2}\underline{V}\,\underline{I}^* = -(S_1 + S_2) = -(1.38\angle -22.0°)$ VA

→ power developed by source : $-S_{source} = (1.38\angle -22°)$ VA

power factor of load :

$pf = \cos(-22°) = 0.927$

Problem 2:

Suppose that s-domain analysis has given you:

$$V(s) = V_m \frac{s^2}{(s+R/L)(s^2+\omega^2)}$$

Determine $v(t)$.

$$\frac{1}{V_m} V(s) = \frac{s^2}{(s+R/L)(s+j\omega)(s-j\omega)}$$

$$= \frac{K_1}{s+R/L} + \frac{K_2}{s+j\omega} + \frac{K_2^*}{s-j\omega}$$

$$K_1 = \left.\frac{s^2}{s^2+\omega^2}\right|_{s=-R/L} = \frac{(R/L)^2}{(R/L)^2+\omega^2}$$

$$K_2 = \left.\frac{s^2}{(s+R/L)(s-j\omega)}\right|_{s=-j\omega} = \frac{-\omega^2}{(-j\omega+R/L)(-2j\omega)}$$

$$= \frac{j\omega}{2(-R/L+j\omega)} = \frac{1}{2}\frac{j\omega}{(R/L)^2+\omega^2}\left(-\frac{R}{L}-j\omega\right)$$

$$= \frac{1}{2}\frac{\omega^2-j\omega R/L}{(R/L)^2+\omega^2} = \frac{1}{2}\frac{1-jR/\omega L}{1+(R/\omega L)^2} = \frac{1}{2}\frac{e^{j\phi}}{\sqrt{1+(R/\omega L)^2}}$$

where $\phi = -\tan^{-1}\left(\frac{R}{\omega L}\right)$

$$\longrightarrow \quad v(t) = \frac{(R/\omega L)^2 V_m}{1+(R/\omega L)^2} e^{-\frac{R}{L}t} + \frac{1}{2}\frac{V_m}{\sqrt{1+(R/\omega L)^2}} \times$$

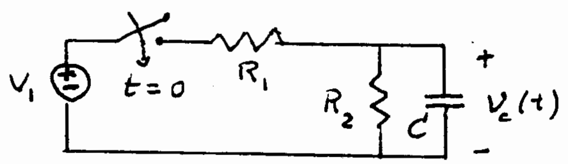
$$\times \left[e^{j\phi}e^{-j\omega t} + e^{-j\phi}e^{j\omega t}\right]$$

$$v(t) = V_m \frac{(R/\omega L)^2}{1+(R/\omega L)^2} e^{-\frac{R}{L}t} + \frac{V_m}{\sqrt{1+(R/\omega L)^2}}\cos(\omega t-\phi)$$

Problem 3:
In the circuit below the switch was open for a long time. Analyze the circuit in the *s*-domain.

A) Find $V_c(s)$.

B) Use the intial and final value theorems to find $v_c(0^-)$ and $v_c(\infty)$.



$$Z_2 = R_2 \parallel \frac{1}{sC'} = \frac{R_2/sC'}{R_2 + \frac{1}{sC'}} = \frac{R_2}{sR_2C' + 1}$$

A]

$$V_c(s) = \frac{Z_2}{Z_2 + R_1}(V_1/s) = \frac{V_1}{s}\,\frac{1}{1 + \frac{R_1}{Z_2}}$$

$$= \frac{V_1}{s}\,\frac{1}{1 + \frac{(sR_2C+1)R_1}{R_2}}$$

$$= \frac{V_1}{s}\,\frac{R_2}{R_2 + sR_2R_1C + R_1} = \frac{V_1}{s}\,\frac{R_2}{sR_2R_1C' + R_1 + R_2}$$

B]

$$v_c(0^-) = \lim_{s\to\infty}(s V_c(s))$$

$$= \lim_{s\to\infty}\left(V_1 \frac{R_2}{sR_2R_1C + R_1 + R_2}\right) = 0$$

$$v_c(\infty) = \lim_{s\to 0}(s V_c(s))$$

$$= \lim_{s\to 0}\left(V_1 \frac{R_2}{sR_2R_1C' + R_1 + R_2}\right)$$

$$= V_1 \frac{R_2}{R_1 + R_2}$$