

Problem 1

$$R_1 = 20 \Omega$$

$$R_2 = 12 \Omega$$

$$C = 125 \mu F$$

$$L = 25 \mu H$$

$$A.) \quad Z_1 = \left(R_1 \parallel \frac{1}{sC} \right) = \frac{R_1}{sR_1C + 1}$$

Node analysis: $\frac{V_1}{Z_1} + \frac{V_1 - V_o}{R_2} = \bar{I}_g$

$$\frac{V_o - V_1}{R_2} + \frac{V_o}{sL} = 0 \rightarrow V_1 = V_o \left(\frac{R_2}{sL} + 1 \right)$$

$$\rightarrow V_o \left[\underbrace{\frac{R_2}{sLZ_1} + \frac{1}{Z_1} + \frac{1}{sL} + \cancel{\frac{1}{R_2}} - \cancel{\frac{1}{R_2}}}_{\frac{R_2 + sL + Z_1}{sLZ_1}} \right] = \bar{I}_g$$

$$\rightarrow H(s) = \frac{V_o(s)}{\bar{I}_g(s)} = \frac{sLZ_1}{sL + R_2 + Z_1} = \frac{sL}{sL/Z_1 + R_2/Z_1 + 1}$$

$$= \frac{sLR_1}{sL(sR_1C + 1) + R_2(sR_1C + 1) + R_1}$$

$$= \frac{sLR_1}{s^2LR_1C + s(L + R_2R_1C) + R_1 + R_2}$$

$$= \frac{5 \times 10^{-4} s}{6.25 \times 10^{-11} s^2 + 5.5 \times 10^{-5} s + 32}$$

$$H(s) = \frac{8 \times 10^6 s}{s^2 + 8.8 \times 10^5 s + 5.12 \times 10^{11}} \Omega$$

B.)

$$\tilde{H}(s) = Z_1(s) = \frac{20}{2.5 \times 10^{-6} s + 1} = \frac{8 \times 10^6}{s + 4 \times 10^5} \Omega$$

Problem 2

$$H(s) = \frac{s}{s^2 + 10s + 34}$$

A.) $\omega = 3 \text{ rad/s}$ $v_i(t) = 5 \cos(3t) \text{ V}$

$$H(j\omega) = \frac{3j}{-9 + 30j + 34} = \frac{3j}{25 + 30j}$$

$$|H(j\omega)| = \frac{3}{\sqrt{25^2 + 30^2}} = 7.68 \times 10^{-2}$$

$$\theta(j\omega) = \frac{\pi}{2} - \tan^{-1}\left(\frac{30}{25}\right) = 0.695 \hat{=} 39.8^\circ$$

→ steady state $v_o(t) = \underline{0.384 \cos(3t + 39.8^\circ) \text{ V}}$

B.) $v_i = 5\delta(t) \text{ V}$

→ $v_o(t) = 5h(t)$

$$s^2 + 10s + 34 = 0 \quad \rightarrow \quad s = -5 \pm \sqrt{25 - 34}$$
$$(s + 5 - 3j)(s + 5 + 3j) = 0 \quad = -5 \pm j3$$

$$H(s) = \frac{A}{s + 5 - 3j} + \frac{A^*}{s + 5 + 3j}$$

$$A = \frac{-5 + 3j}{-5 + 3j + 5 + 3j} = \frac{-5 + 3j}{6j} = \frac{1}{2} + \frac{5}{6}j = 0.972e^{j59^\circ}$$

$$\rightarrow h(t) = \left(\frac{1}{2} + \frac{5}{6}j\right) e^{-5t} e^{3jt} + \left(\frac{1}{2} - \frac{5}{6}j\right) e^{-5t} e^{-3jt}$$
$$= 1.44 e^{-5t} \cos(3t + 59^\circ)$$

$$\underline{v_o(t) = 9.72 e^{-5t} \cos(3t + 59^\circ) \text{ V}}$$

Problem 3

$$h_{11} = \frac{V_1}{I_1} \Big|_{V_2=0} = Z_i$$

$$h_{12} = \frac{V_1}{V_2} \Big|_{I_1=0} = 0$$

$$h_{21} = \frac{I_2}{I_1} \Big|_{V_2=0} = \beta$$

$$h_{22} = \frac{I_2}{V_2} \Big|_{I_1=0} = \frac{1}{Z_o}$$