Experiment 4: Dynamic parameter identification

Objective:
1. To study the torsional control system (Model 205a)
2. To perform the system identification

References:
1. ECP systems manual (Model 205a)

Apparatus:
1. Torsion control model 205a
2. PC
3. Control Box
4. Disks and brass weights

Prelab Report:
1. What is a Torsion control system? What it is comprised of? Give a brief summary of what the experiment is all about.
2. What are the application areas where Torsion control system is important?

Postlab Report:
- Answer the questions asked within or at the end of the procedure
6.1 System Identification

This section gives a procedure for identifying the plant parameters applicable to Eq's (5.1-1 through 5.1-8). The approach will be to use certain fundamental properties of lightly damped second order systems to indirectly measure the inertia, spring, and damping constants of the plant by making measurements of the plant while set up in a pair of classical spring-mass configurations.

Procedure:

1. For Model 205a, clamp the center disk to put the mechanism in the configuration shown in Figure 6.1-1a using the 1/4" bolt, square nut, and clamp spacer. Only light torquing on the bolt is necessary. For Model 205 (2 disk) use one half of the lower disk to clamp the shaft at the center location.

2. Secure four 500g masses on the upper and lower disks (upper disk only for Model 205) as shown in the figure. Verify that the masses are secured per Section 2.2.2 and that each is at a center distance of 9.0 cm from the shaft center-line.

3. With the controller powered up, enter the Control Algorithm box via the Set-up menu and set $T_s = 0.00442$. Enter the Command menu, go to Trajectory and select Step, Set-up. Select Open Loop Step and input a step size of 0 (zero), a duration of 4000 ms and 1 repetition. Exit to the Background Screen by consecutively selecting OK. This puts the controller board in a mode for acquiring 8 sec of data on command but without driving the actuator. This procedure may be repeated and the duration adjusted to vary the data acquisition period.
4. Go to Set up Data Acquisition in the Data menu and select Encoder #1 and Encoder #3 (for Model 205a, Encoder #1 and Encoder #2 for Model 205) as data to acquire and specify data sampling every 2 (two) servo cycles, i.e. every 2 $T_s$'s. Select OK to exit. Select Zero Position from the Utility menu to zero the encoder positions.

5. Select Execute from the Command menu. Prepare to manually displace the upper disk approximately 20 deg. Exercise caution in displacing the inertia disk; displacements beyond 40 deg may damage and possibly break the flexible drive shaft. (Displacements beyond 25 deg will trip a software limit which disables the controller indicated by "Limit Exceeded" in the Controller Status box in the Background Screen. To reset, simply reselect Execute from the Command menu.) With the upper disk displaced approximately 20 deg ($\leq 1000$ encoder counts as read on the Background Screen display) in either direction, select Run from the Execute box and release the disk approximately 1 second later. The disk will oscillate and slowly attenuate while encoder data is collected to record this response. Select OK after data is uploaded.
6. Select Set-up Plot from the Plotting menu and choose Encoder #3 position (for Model 205a, or Encoder #2 for Model 205); then select Plot Data from the Plotting menu. You will see the upper disk time response.

7. Choose several consecutive cycles (say 5 to 10) in the amplitude range between 100 and 1000 counts (This is representative of oscillation amplitudes during later closed loop control maneuvers. Much smaller amplitude responses become dominated by nonlinear friction effects and do not reflect the salient system dynamics) Divide the number of cycles by the time taken to complete them being sure to take beginning and end times from the same phase of the respective harmonic cycle. Convert the resulting frequency in Hz to radians/sec. This *damped frequency*, $\omega_d$, approximates the *natural frequency*, $\omega_n$, according to:

$$\omega_{d31} = \frac{\omega_{d31}}{\sqrt{1 - \zeta_{d31}^2}} \approx \omega_{d31} \quad \text{(for small } \zeta_{d31} \text{)}$$

where the "d31" subscript denotes disk #3, trial #1. (Close the graph window by clicking on the left button in the upper right hand corner of the graph. This will collapse the graph to icon form where it may later be brought back up by double-clicking on it.)

8. Remove the four masses from the third (upper) disk and repeat Steps 5 through 7 to obtain $\omega_{nd32}$ for the unloaded disk. If necessary, repeat Step 3 to reduce the execution (data sampling) duration.

9. Measure the reduction from the initial cycle amplitude $X_0$ to the last cycle amplitude $X_n$ for the $n$ cycles measured in Step #8. Using relationships associated with the *logarithmic decrement*:

$$\frac{\zeta_{d32}}{\sqrt{1 - \zeta_{d32}^2}} = \frac{1}{2\pi n} \ln \left( \frac{X_0}{X_n} \right) \rightarrow \zeta_{d32} = \frac{1}{2\pi n} \ln \left( \frac{X_0}{X_n} \right) \quad \text{(for small } \zeta_{d32} \text{)}$$

find the damping ratio $\zeta_{d32}$ and show that for this small value the approximations of Eq’s (6.1-1, -2) are valid.

10. Repeat Steps 5 through 9 for the lower disk, disk #1. Here in Step 6 you will need to remove Encoder #3 position (#2 for Model 205) and add Encoder #1 position to the plot set-up. Hence obtain $\omega_{nd11}$, $\omega_{nd12}$ and $\zeta_{d12}$. How does this damping ratio compare with that for the upper disk?

---

1You may “zoom” the plot via Axis Scaling for more precise measurement in various areas. For an even greater precision, the data may be examined in tabular numerical form – see Export Raw Data, Section 2.1.7.3.
11. Use the following information pertaining to each mass piece to calculate the portion of each disk's inertia attributable to the four masses for the "d31" and "d11" cases.

- Mass (incl. bolt & nut) = 500g (± 5g)
- Dia = 5.00 cm (± 0.02 cm)

12. Calling this inertia $J_m$ (i.e. that associated with the four masses combined), use the following relationships to solve for the unloaded disk inertia $J_{d3}$, and upper torsional shaft spring $k_{d3}$.

- $k_{d3}/(J_m+J_{d3}) = (\omega_{nd31})^2$  
- $k_{d3}/J_{d3} = (\omega_{nd32})^2$

Find the damping coefficient $c_{d3}$ by equating the first order terms in the equation form:

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + c/Js + k/J$$

Repeat this for the lower unloaded disk inertia (this includes the reflected inertias of the motor, belt, and pulleys), spring and damping $J_{d1}$, $c_{d1}$ and $k_{d1}$ respectively.\[2\]

Now all dynamic parameters have been identified! Values for $J_1$ and $J_2$ for any configuration of masses may be found by adding the calculated inertia contribution of the masses to that of the unloaded disk.\[3\]

The following is necessary to establish the hardware gain for control modeling purposes.

Procedure:

13. Remove the entire upper disk, unfasten the mid-shaft disk and clamp and replace the four masses on the lower disk (only) at the 9.0 center distance following the guidelines of Step 2. Verify that the masses are secure and that the disk rotates freely. Hook up the drive power to the mechanism.

14. In the Trajectory window deselect Unidirectional moves (i.e. enabling bi-directional inputs) select Step, Set-up. Choose Open Loop Step, and input 1.00 Volts, 500 ms, 2 reps. Execute this open loop step via the Execute menu. (For higher open loop voltages, this move may trip a software speed limit which disables the controller indicated by "Limit Exceeded" in the Controller Status box in the "desk top". Again, to reset, simply reselect Execute from the Execute menu.)

\[2\]Steps 11 and 12 may be done later, away from the laboratory, if necessary.

\[3\]In plant configurations where a disk is used in the center location, the inertia and damping parameters may be assumed to be the same as for the upper disk.
Chapter 6. Experiments

menu.) Go to Set-up in the Plot Data menu and select Encoder #1 velocity for plotting.

15. Plot this data and observe four velocity profile segments with nominal shapes of: linear increase (constant acceleration), constant (zero acceleration), linear decrease (deceleration), and constant. Obtain the acceleration, $\dot{\theta}_{1e}$, (counts/s$^2$) by carefully measuring the velocity difference and dividing by the time difference (500 ms) through the positive-sloped linear segment. Repeat this for the negative-sloped segment. Calculate the average magnitude of the positive and negative accelerations for use in obtaining $k_{hw}$ below.

16. Save any files or plots of interest. Exit the executive program and power down the system.

Transfer Function Calculation

The so-called hardware gain, $k_{hw}$ of the system is comprised of the product:

$$k_{hw} = k_ck_akt_pk_ek_s$$

where:

- $k_c$, the DAC gain, = $10V/32,768$ DAC counts
- $k_d$, the Servo Amp gain, = approx. 2 (amp/V)
- $k_t$, the Servo Motor Torque constant = approx. 0.1 (N-m/amp)
- $k_p$, the Drive Pulley ratio = 3 (N-m @ disk / N-m @ Motor)
- $k_e$, the Encoder gain, = 16,000 pulses / $2\pi$ radians
- $k_s$, the Controller Software gain, = 32 (controller counts / encoder or ref input counts)

In Step 15, we obtained the acceleration $\dot{\theta}_{1e}$ (counts/s$^2$) of a known inertia, $J_1=J_m+J_{d1}$ with a known voltage applied at the DAC. Thus by neglecting the relatively small friction:

---

4 Some small deceleration will exist due to friction.
5 For more precise measurement you may "zoom in" on this region of the plot using Axis Scaling in the Plotting menu.
6 It is possible to read the accelerations directly by plotting Encoder #1 acceleration. This data, obtained by double numerical differentiation, is typically somewhat noisy however. The student may want to verify this by observing the acceleration plot.
7 It contains software gain also. This software gain, $k_s$, is used to give higher controller-internal numerical resolution and improves encoder pulse period measurement for very low rate estimates.
8 The “controller counts” are the counts that are actually operated on in the control algorithm. i.e. The system input (trajectory) counts and encoder counts are multiplied by 32 prior to control law execution.
\[
\text{Applied Torque} = J_1 \dot{\theta}_1 = J_1 \ddot{\theta}_1 e / k_e 
\]  
\hspace{1cm} (6.1-7)

and we have a direct measurement of the four-term product \( k_d k_l k_e k_p \). i.e.:

\[
1.00V \, k_d k_l k_e = \text{Applied Torque in Step 15} 
\]  
\hspace{1cm} (6.1-8)

Use (6.1-6 through 6.1-8) to solve for \( k_{hw} \) using the specified values for \( k_c \) and \( k_s \).

For control purposes it is generally desirable to put the transfer function in denominator-monic form (leading term in \( D(s) \) has unity coefficient).

Construct a denominator-monic plant model suitable for control design for the case of two 500g weights on each of the upper & lower disks with each mass centered at 9.0 cm from the shaft center-line – see Figure 6.1-1b. Use the results of this section and equations (5.1-4 through 5.1-6) to generate these transfer functions where the hardware gain multiplies the numerator for each transfer function.

**Questions / Exercises:**

A. Report the measured properties and derived parameter values:
\( J_m, J_{d1}, J_{d2}, J_{d3}, c_{d1}, c_{d2}, c_{d3}, k_{d1}, k_{d2}, \text{ and } k_{hw} \)

B. Construct a denominator-monic plant model suitable for control design for the case of two 500g weights on the upper most & lower disks with each mass centered at 9.0 cm from the shaft center-line – see Figure 6.1-1b. Use the results of this section and equations (5.1-4 through 5.1-6) to generate these transfer functions where the hardware gain multiplies the numerator for each transfer function.

C. What are the units of \( k_{hw} \)?